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Excitation of the Magnetospheric Cavity by Space-Based ELF/VLF Transmitters

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14. ABSTRACT
During the period of performance, Stanford University completed the development of integral equations describing the distribution of current along a dipole antenna radiating ELF/VLF waves in the magnetospheric cavity. It was found that the radiation resistance was much smaller near the lower-hybrid resonance frequency than previously believed. Stanford University included the effects of ion temperature in the plasma dielectric tensor. It was found that the ion temperature had a first-order effect upon the distribution of ELF/VLF waves in the magnetospheric cavity.

15. SUBJECT TERMS
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1. SUMMARY

During the period of performance (10/31/04 - 10/30/05), Stanford University completed the development of an analytical model describing the distribution of current along a dipole antenna radiating ELF/VLF waves in the magnetospheric cavity. It was found that for an antenna perpendicular to the Earth's magnetic field, the predicted radiation resistance of the antenna was a strong function of frequency near the lower-hybrid-resonance frequency. In addition, it was found that the finite temperature of the thermal ions within the magnetospheric cavity appeared to have a significant effect upon the distribution of ELF/VLF waves radiated by space-based ELF/VLF transmitters within the cavity.

2. CONTRACT PURPOSE

The overall objectives of this contract are to determine the following: 1) the optimum orbit for exciting the cavity resonance by a space-based ELF/VLF transmitter, 2) the antenna type and configuration necessary to excite various cavity modes with the radiated ELF/VLF waves, 3) the effects of Landau damping on the ELF/VLF waves within the cavity and examine possible methods of minimizing this damping, 4) the effectiveness of the radiated ELF/VLF cavity waves in precipitating energetic radiation belt particles, and 5) the optimum spacecraft orbit, antenna configuration, and ELF/VLF transmitter frequency spectrum for precipitating energetic radiation belt particles over a wide range of energies.

3. PERIOD OF PERFORMANCE

The period of performance for this report extended from October 31, 2004, through October 30, 2005.

4. WORK PROVIDED

During the period of performance, Stanford University: 1) Completed the development of analytical models describing the current distribution of dipole antennas used to radiate ELF/VLF waves from spacecraft within the magnetosphere, and 2) Initiated a study of the effect of the temperature of thermal ions within the magnetospheric cavity upon the distribution of ELF/VLF waves radiated by space-based ELF/VLF transmitters within the cavity.

5. RESULTS

5.1. The Radiation Resistance of a Perpendicular Dipole Antenna

The integral equation for the dipole antenna current distribution for the case in which the dipole antenna is perpendicular to the Earth's magnetic field B_o has the form:

$$\frac{\mu_o \gamma}{8\pi^2} \int_0^{2\pi} \int_{-h}^{h} \frac{e^{-\beta_c \gamma R_a(x,x')}}{R_a(x,x')} I(x')dx'd\phi = \frac{\beta_p}{\omega}(b_o \cosh\beta_p x - \frac{1}{2} V_o \sinh\beta_p |x|) \quad (1)$$

where $\beta_p = \sqrt{|P|}\omega/c, \gamma = \sqrt{|P/S|}$, h is the antenna half-length, P and S are plasma parameters defined in *Stix*[1992], b_o is determined by the condition that the current vanishes at the dipole end points, and:

$$R_a(x,x') = [(x - x')^2 + (a \sin\phi)^2 - (a \cos\phi)^2/\gamma^2]^{1/2}$$

where it is assumed that the antenna lies along the x axis, a is the antenna radius, x is the observation point, and x' is the source point.

It can be seen that the right-hand side of (1) consists of evanescent waves which decay exponentially along the antenna. Thus, to first order, the current will also decay exponentially along the antenna as $e^{-\beta_p x}$. In this case, the current moment will increase only marginally if the antenna length is increased beyond the value $h \simeq 1/\beta_p$. At an altitude of 6000 km near the magnetic equatorial plane, $1/\beta_p \simeq 100$ m. At an altitude of 600 km in LEO, $1/\beta_p \simeq 30$ m. Thus, the appropriate antenna length is a function of spacecraft altitude. If we assume that the antenna is short enough that $(\beta_p h)^2 \ll 1$, then the current distribution will be triangular and (1) becomes:

$$\frac{\mu_o \gamma}{8\pi^2} \int_0^{2\pi} \int_{-h}^{h} K_d(x, x^{'}) I(x^{'}) dx^{'} d\phi = \frac{\beta_p^2 V_o}{2\omega}(h - |x|) \tag{2}$$

where:

$$K_d(x, x^{'}) = \int_0^{2\pi} [\frac{e^{-\beta_c \gamma R_a(x,x^{'})}}{R_a(x, x^{'})} - \frac{e^{-\beta_c \gamma R_a(h,x^{'})}}{R_a(h, x^{'})}] \frac{d\phi}{2\pi} \tag{3}$$

Equation (2) can be solved for the current distribution $I(x)$ using standard methods [*King et al.*, 2002]. Once the current distribution is found, the radiation resistance R_r of the dipole antenna can be calculated. It can be shown that the first order solution for the radiation resistance has the form:

$$R_r = \frac{Z_o \gamma}{2\pi \beta_p h} \int_{-h}^{h} (1 - |x^{'}|/h)\ K_{dr}(0, x^{'})\ dx^{'} \tag{4}$$

where $K_{dr}(0, x^{'})$ is the real part of $K_d(0, x^{'})$.

The radiation resistance of a dipole antenna in a magnetized plasma has been calculated in the past using the quasi-static theory [*Balmain*, 1964]. This theory assumes that wave phase shifts along the antenna are negligible and that the radiation resistance can be calculated from a scalar potential for frequencies above the lower-hybrid-resonance frequency. This model is still in use after four decades [e.g., *Chugunov et al.*, 2003], but its accuracy has not been previously assessed in a meaningful way. For a dipole antenna with a triangular current distribution oriented perpendicular to B_o, the predicted radiation resistance for the quasi-static model has the value:

$$R_{rqs} = \frac{Z_o \gamma}{\pi \beta_p h |P|} [\log(\frac{2h}{a}) - 1] \tag{5}$$

On the other hand, in the integral equation method, as the wave frequency approaches the lower-hybrid-resonance frequency, the parameter γ becomes arbitrarily large, and (4) can be evaluated analytically to yield:

$$R_r = \frac{Z_o \gamma}{\pi \beta_p h |P|} [\log(\frac{2h}{a}) - \log(\gamma \beta_c h)] \tag{6}$$

With the aid of (5) and (6), we can find the ratio of R_r to R_{rqs} :

$$\frac{R_r}{R_{rqs}} = \frac{[\log(\frac{2h}{a}) - \log(\gamma \beta_c h)]}{[\log(\frac{2h}{a}) - 1]} \tag{7}$$

It can be seen from (7) that as the wave frequency approaches the lower-hybrid-resonance frequency and γ becomes large, the radiation resistance predicted by the integral equation method R_r will become much smaller than that predicted by the quasi-static model R_{rqs}. This is an important finding, since the spacecraft-based ELF/VLF transmitters used to excite the magnetospheric cavity will operate at frequencies close to the lower-hybrid-resonance frequency, and the optimum transmitter system design requires good knowledge of the antenna radiation resistance. Further work on this topic will be performed during the next reporting period.

5.2. The Effect of Ion Temperature upon the Distribution of ELF/VLF Waves Within the Magnetospheric Cavity.

ELF/VLF waves injected from a spacecraft in the inner magnetosphere distribute power throughout the radiation belts as a function of injection frequency and wave normal angle. In a cold plasma, these waves reflect in the cavity at the points at which the wave frequency is equal to the lower-hybrid-resonance frequency. This reflection occurs due to the effects of the local ions upon the propagation characteristics of the waves. Due to the ions, the refractive index surface transitions from an open surface to a closed surface as the wave propagates downward and the ratio of the wave frequency to the local lower-hybrid-resonance frequency becomes less than unity. As soon as the wave reaches an altitude at which the refractive index surface is closed, the reflection process begins.

Since the ions play such an important role in the reflection process for the waves, it is important to determine if the reflection process might change if the ion temperature was taken into consideration in the plasma dielectric tensor. In this regard, we have found that the ion temperature plays a much larger role than expected. Figure 1 shows an example of the results we have obtained. This figure shows one quadrant of the cross section of the refractive index surface of a VLF wave of 3.5 kHz frequency. It is assumed that the wave is located near $L = 2$ where the local plasma frequency is 500 kHz and the local electron gyrofrequency is 100 kHz. The local lower-hybrid-resonance frequency is 2.5 kHz. If it is assumed that the ions are cold, then the refractive index surface should be open, since the wave frequency is larger than the lower-hybrid-resonance frequency. This surface is shown as the dashed line in Figure 1. On the other hand, if it is assumed that the proton temperature is 1 eV, a common value near $L = 2$, it is found that the refractive index surface is now closed, as shown by the solid line in Figure 1.

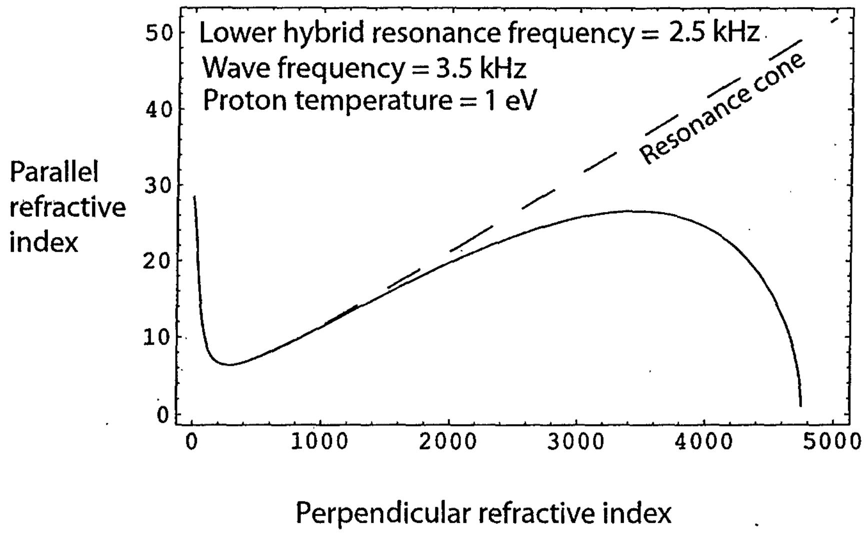

Figure 1. Cross section of the ELF/VLF wave refractive index surface for the two cases of cold ions (dashed line) and ions with a temperature of 1 eV (solid line). It is assumed that the wave frequency is 3.5 kHz and the lower-hybrid-resonance frequency is 2.5 kHz. It can be seen that the inclusion of ion temperature causes the refractive index surface to change from an open surface to a closed surface.

The fact that the wave refractive index surface is closed in Figure 1 for an proton temperature of 1 eV, suggests that the wave would have reflected at some altitude above the point for which the surface was plotted. Thus the inclusion of ion temperature has the effect of raising the reflection points of the ELF/VLF waves in the magnetospheric cavity. Inclusion of ion temperature also has the effect of altering the general propagation paths of the waves since the refractive index surfaces of the waves undergo large changes. Thus we conclude that for the frequencies of interest, the effects of the finite temperature of the ions must be included in the raytracing code if we are to accurately describe the distribution of wave power within the magnetospheric cavity. Further work on this topic will be carried out during the next reporting period.

6. LIST OF PERSONNEL CONTRIBUTING TO REPORT

The scientists and engineers of Stanford University who contributed to the work reported in this document are as follows: Tim Bell, Umran Inan, and P. Kulkarni.

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